(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = t\int_0^t e^{-2u} sin4u\, du.$ 5

b) Show that the set of functions $\sin nx, n = 1,2,3 \ldots \ldots$ is orthogonal on $(0,2\pi)$. 5

c) Calculate Spearman's rank correlation coefficient *R*, from the given data, 5

X:	12,	17,	22,	27,	32.
Y:	113,	119,	117,	115,	121

d) Find the constants a, b, c, d, e if

$$f(z) = ax^3 + bxy^2 + 3x^2 + cy^2 + x + i(dx^2y - 2y^3 + exy + y)$$

is analytic. 5

2. a) Find Laplace transform of the periodic function, defined as

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases} \text{ and } f(t+2) = f(t) \text{ for } t > 0$$ 6

b) If $v = 3x^2y + 6xy - y^3$, show that *v* is harmonic and find the corresponding analytic function $f(z) = u + iv$. 6

c) Obtain Fourier series of $f(x) = x^2$ in $(0,2\pi)$. Hence, deduce that – 8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + + \cdots$$

3. a) Using convolution theorem, find the inverse Laplace transform of 6

$$F(s) = \frac{1}{s^2(s+5)^2}$$

b) Solve $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$, subject to the conditions,
$u(0,t) = 0, u(1,t) = 3t, u(x,0) = 0,\quad 0 \le x \le 1,$ taking $h = 0.25$
up to 3 seconds only by using Bender –Schmidt method. 6

c) Using Residue theorem, evaluate,

i) $\int_0^{2\pi} \frac{d\theta}{17-8cos\theta}$ ii) $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$ 8

[TURN OVER]

4. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0,t) = 0,\ u(1,t) = 0, u(x,0) = 100(x - x^2)$, with $h = 0.25$ for one-time step. 6

b) Evaluate $\int_c \frac{z}{(z-2)(z+1)^2} dz$, $C: |z| = 3$. 6

c) Solve $(D^2 - 2D + 1)y = e^{-t}$ with $y(0) = 2$, $y'(0) = -1$ where $D \equiv \frac{d}{dt}$ 8

5. a) Obtain all possible Taylor's and Laurent series which represent the function

$f(z) = \frac{z}{z^2 - 5z + 6}$ indicating the region of convergence. 6

b) Evaluate $\int_0^\infty te^t cos^2 t\, dt$ 6

c) Obtain half range Fourier cosine series of $f(x) = x(\pi - x), 0 < x < \pi$.
Using Parseval's identity, deduce that – 8

$$\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots\cdots$$

6. a) Find the image of the circle $|z| = 2$ under the transformation $w = z + 3 + 2i$.
Draw the sketch. 6

b) A rectangular metal plate with insulated surfaces of width l and so long as compared to its breadth that it can be considered infinite in length without introducing an appreciable error. If the temperature along one short edge $y = 0$ is given by $u(x,0) = u_0 \sin\left(\frac{\pi x}{l}\right)$ for $0 < x < l$ and other long edges $x = 0$ and $x = l$ and the short edges are kept at zero degrees temperature, find the function $u(x,y)$ describing the steady state, assuming that in the steady state the heat distribution function $u(x,y)$ satisfies the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$. 6

c) Production (in metric kiloton) of wheat in a country is given by the following data,

Year (x)	2005	2007	2009	2011	2013	2015	2017
Production (y)	8	12	15	19	21	22	25

Fit a straight line to the data and estimate the production in the year 2010. 8
