Q.P. Code : 536601

(3 Hours) **[Total Marks : 80**

N.B. (1) Questions **No.1** is **compulsory**.
(2) Attempt **any three** questions out of the remaining **five** questions.
(3) **Figures** to the **right** indicate **full marks**.

1. (a) Find the Laplace Transform of $t\ e^{3t}\ \sin 4t$. **5**

(b) Apply Cayley-Hamilton theorem for the matrix A & hence find $A^8 = 625\ I$, where $A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$. **5**

(c) Evaluate $\int_c \bar{z}\ dz$, where c is the upper half of the circle $r = 1$. **5**

(d) With usual notation find p of Binomial distribution if $n = 6$, $9P\ (x = 4) = P(x = 2)$. **5**

2. (a) Find the analytic function whose imaginary part is $v = 3x^2y + 6xy - y^3$ show that v is harmonic. **6**

(b) Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t}\, dt$. **6**

(c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable & find the diagonal matrix & the transforming matrix. **8**

3. (a) Find inverse Laplace Transform of $\frac{(s+2)^2}{(s^2 + 4s + 8)^2}$ By convolution theorem. **6**

(b) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory. **6**

(c) Using the Kunh - Tucker conditions solve the following N.L.P.P. **8**

Maximise : $z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to : $x_1 + 3x_2 \leq 6$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$

TURN OVER

4. (a) Find the Bilinear transformation which maps the points $z = 1, -i, 2$ onto the Points $w = 0, 2, -i$. **6**

(b) Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$. **6**

(c) Using Lagrangian multiplier method solve the following N.L.P.P. **8**

Optimise : $z = 6x_1^2 + 5x_2^2$

Subject to : $x_1 + 5x_2 = 7,\ x_1, x_2 \geq 0$

5. (a) Find the eigen values & eigen vectors for $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$. **6**

(b) Evaluate $\int_c \frac{z^2}{(z-1)^2 (z+1)} dz$ where c is $|z| = 2$. **6**

(c) Find : **8**

(i) $L^{-1}\left[\frac{s+2}{s^2 - 4s + 13}\right]$

(ii) $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$.

6. (a) Calculate Spearman's coefficient of rank correlation from the following data : **6**

x	12	17	22	27	32
y	113	119	117	115	121

(b) Find the residues of $f(z) = \frac{\sin \pi z}{(z-1)^2 (z-2)}$ at its poles. **6**

(c) Reduce the following quadratic form to canonical form. Also find its rank & signature **8**

$21x_1^2 + 11x_2^2 + 2x_3^2 - 8x_2 x_3 + 12x_3 x_1 - 30x_1 x_2$.