S.E-IV Sem Biotech.
Applied maths-IV
BT/II

BT/IV/CBGS/AM-IV

2115115



QP Code: 3665

(3 Hours)

[Total Marks :80



- (2) Attempt any three questions from the remaining
- (3) Figures to the right indicate full marks.
- 1 (a) Obtain half range cosine series for $f(x) = x(\pi-x)$, $0 < x < \pi$.
 - (b) Find the total work done in moving a particle in the force field
 F = 3xy i 5 z j + 10x k along x = t²+1,
 y = 2 t², z = t³ from t=1 and t =2.
 - (c) Prove that

$$\int_{A}^{B} (2xy^{3} - y^{2} \cos x) dx + (1 - 2y \sin x + 3x^{2}y^{2}) dy = \frac{\pi}{4}$$

- (d) Show that the set of functions cosx, cos 2π , cos 3π is a set of orthogonal functions over $(-\pi, \pi)$, Hence, construct a set of orthonormal functions.
- 2. (a) Find the Fourier expansion of $f(x) = 2x-x^2$, $0 \le x \le 3$ whose period is 3.
 - (b) Evaluate $\iint_{S} (\nabla x \overline{F}) . d\overline{s}$ where $\overline{F} = (2y^2 + 3z^2 x^2) i + (2z^2 + 3x^2 y^2) j + (2x^2 + 3y^2 z^2)k$
 - over the part of the sphere $x^2+y^2+z^2$. 2ax + az = 0 cut off by the plane z = 0(c) A string is stretched between x = 0 and x = 1 and both ends given a displacement $y = a\sin pt$ perpendicular to the string. If the string satisfies the differntiel
 - equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}$, show that the oscillation of the string are given by

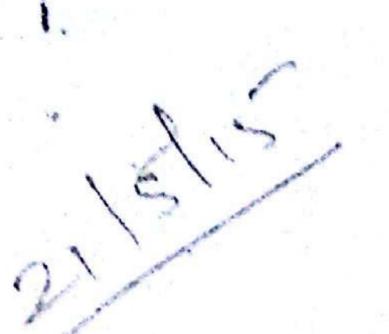
$$y = a \sec \frac{p\ell}{2c} \cos \left(\frac{px}{c} - \frac{p\ell}{2c}\right) \sin pt.$$

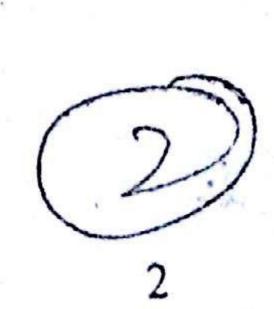
3. (a) A tightly stretched string with fixed ends x=0 and x = ℓ and initially in a position $y = a \sin^3(\pi x / \ell)$ is released from the position of rest. Find the displacement of any point at any time if y, the vertical displacement satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

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3. (b) Using Fourier cosine integral, prove that

$$e^{-x}\cos x = \frac{2}{\pi} \int_{0}^{\infty} \left(\frac{\omega^{2} + 2}{\omega^{2} + 4}\right) \cos \omega x \ d\omega$$

(c) Solve the equation $\frac{\partial u}{\partial t} = \frac{K\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod of length

e subject to the following conditions.

- (i) u is not infinitely for $t \to \infty$
- (ii) $\frac{\partial u}{\partial x} = 0$ for x = 0 and $x = \ell$ for any time t.
- (iii) $u = \ell x x^2$ for t = 0 between x = 0 and $x = \ell$.

4. (a) Evaluate
$$\iint_{S} x^{2} dy dz + y^{2} dz dx + 2z(xy - x - y) dx dy$$

where S is the surface of the cube bounded by

$$x = 0, x = 1, y = 1, z = 0, z = 1.$$

- (b) Obtain Fourier series of xcosx in (-π, π).
- (c) Expand $f(x) = x \sin x$ in the interval $0 \le x \le 2 \pi$.

Hence deduce that $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \frac{3}{4}$

5. (a) Find complex form of Fourier series for cos ax, where a is not an integer is $(-\pi,\pi)$.

(b) A rectangular plate with insulted surface has width of a'cms and so long compared to its width that it may be considered infinite is length without introducing an 6 appreciable error. If the two long edges x=0 and x = a as well as the one short edge are kept at 0°C and the temperature of the other short edge y = 0 is given by

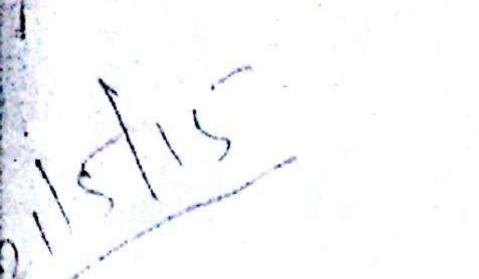
$$u = kx$$
 for $0 \le x \le a/2$
= $k(a-x)$ for $a/2 \le x \le a$

Find the temperature u (x, y) at any point (x, y) of the plate.

(c) Show that $\overline{j} = (2xyz^2) i + (x^2 z^2 + z \cos yz) j + (2x^2 yz + y \cos yz) k is conservative. Find 8$ the scalar potential ϕ such that $\overline{F} = \nabla \phi$ and hence, find the work done by \overline{F} in displacing a particle from A(0,0,1) TOB(1, π /4, 2) along the straight line AB.

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6. (a) Find the Fourier transform of

$$f(x) = 1 + \frac{x}{a}; -a < x < 0$$

$$1 - \frac{x}{a}; 0 < x < a$$

- (b) Using Green's Theorem evaluate $\int_C \overline{F} \cdot d\overline{r}$ where C is the curve enclosing the region bounded by $y^2 = yax$, x = a in the plane z = 0 and $\overline{F} = (2 x^2y + 3 z^2) i + (x^2 + 4y z) j + (2y^2 + 6 xz) k$.
- (c) Find the Fourier expansion of $f(x) = x + x^2$; $-\pi \le x \le \pi$ Hence deduce that

(i)
$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

(ii)
$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

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