Paper / Subject Code: 50501 / Applied Mathematics-III

SE / Biomed / Sem III / Choice Based.

(39)

(16)

Time: 3 Hours

Total Marks: 80

N.B. 1) Question No. 1 is compulsory.

2) Attempt any three out of remaining five Questions.

3) Figure to right indicate full mark.

Q. 1 a) Find the constant a, b, c, d ,e if $F(z) = ax^3+bxy^2+3x^2+cy^2+x+i(dx^2y-2y3+exy+y)$ is Analytic. (05)

b) Find directional derivatives of $\emptyset = 4xy^2 - 2yz^2$ at $[1, -2, , 2]$ in the direction of $2i + 3j + 5k$ (05)

c) Show that the set of functions $Fn(x) = Cosnx$, n = 1,2,3 ....is orthogonal over $[-\pi, \pi]$ Hence construct orthonormal set (05)

d) Find $L[e^{-2t} \; cos2t.cos4t]$ (05)

Q.2 a) Prove that Field $\bar{F} = (z^2 + 2x + 3y)i + (3x + 2y + z)j + (y + 2xz)k$ is irrotational. Find scalarpotentional $\emptyset$ such that $\bar{F} = \nabla\emptyset$ (06)

b) obtain Fourier expansion of $F(x) = 4 - x^2$ in $[0, 2]$ (06)

c) Using Laplace transform Solve the differential equation (08)

$y'' + 2y' - 3y = sint$, Given $y(0)=0$, $y'(0) = 0$

Q.3 a) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{sinx}{x} - cosx\right\}$ (06)

b) Find Complex form of fourier series of $f(x) = e^{3x}$ in $[-\pi, \pi]$ (06)

c) i) Find $L^{-1}\left\{Log\left(1 + \frac{a^2}{s^2}\right)\right\}$ ii) $L^{-1}\left\{\left(\frac{e^{-2s}}{s^2+8s+25}\right)\right\}$ (08)

Q.4 a) Find analytic function f(z) whose imaginary part is $e^x(x\,siny + ycosy) = c$ (06)

b) Find $L^{-1}\left\{\frac{s}{(s^2+4)((s^2+16)}\right\}$ by convolution theorem (06)

c) Find Fourier expansion of $f(x) = 1 + \frac{2x}{\pi}$ $\quad -\pi \le x \le 0$ (08)

$= 1 - \frac{2x}{\pi}$ $\quad 0 \le x \le \pi$ Hence deduce that

$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots\ldots\ldots\ldots$

S.E/Biomed./III/Choice Based/R-2017/AM-III

Q.P. Code: 25477

5. (a) Define Orthogonal set of functions on (a,b) ,Show that the functions $f_1(x) = 5$, $f_2(x) = 3x$ are orthogonal on (-2,2). Determine the constants P , Q such that $f_3(x) = Px^2 + Qx + 9$ is orthogonal to both $f_1(x)$ & $f_2(x)$ on the same interval. 6

(b) Find the analytic function $f(z) = u + iv$ in terms of Z if ,
$4u - 5v = x^3 + x^2 - 3xy^2 - y^2 - 3yx^2 + y^3 - 2xy$. 6

(c) Verify Green's theorem for $\int_C (4xy - x^2)dx + (2x + 6y^2)dy$,
C is the closed curve in the XY-plane bounded by $y = x^2$ and $x = y^2$. 8

6. (a) Find Laplace transform of $f(x) = \begin{cases} \sin 8t & 0 < t < \pi/2 \\ 1 & \pi/2 < t < \pi \end{cases}$ and $f(t) = f(t + \pi)$. 6

(b) Find the invariant points of the Bilinear transformation $w = \left(\frac{4z-9}{z-2}\right)$,also express it in the normal form. 6

(c) Obtain Complex form of Fourier series for $f(x) = \cosh x$ in $(-l, l)$ 8