(3 Hours)   [Total Marks : 80

N.B.   (1) Question No.1 is compulsory.

       (2) Attempt any three questions out of the remaining five questions.

       (3) Figures to right indicate full marks.

1   a)   Evaluate $\int_C (\bar{z} + 2z) dz$ along the circle $x^2 + y^2 = 1$ (5)

    b)   Evaluate the integral using Laplace Transform $\int_0^\infty e^{-t} \left(t\sqrt{1 + \sin t} \right) dt$ (5)

    c)   Determine the analytic function whose real part is $u = -r^3 \sin 3\theta$.   (5)

    d)   A rod of length l has its ends A and B kept at $0^\circ C$ and $100^\circ C$ (5)
respectively until steady state conditions prevail. If the temperature at
B is reduced suddenly to $0^\circ C$ and kept so while that of A is
maintained. Find the temperature $u(x,t)$ at a distance from A and at
time t.

2   a)   Find complex form of Fourier series of $f(x) = e^{2x}$ in (0,2)   (6)

    b)   Find the orthogonal trajectory of the family of curves given by   (6)
$2x - x^3 + 3xy^2 = a$

    c)   Using Bender Schmidt method solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ subject to the   (8)
conditions $u(0,t) = 0$, $u(1,t) = 0$, $u(x,0) = \sin \pi x$, $0 \le x \le 1$.
Assume h=0.2

3   a)   Find k such that $\frac{1}{2}\log(x^2 + y^2) + i \tan^{-1}(\frac{kx}{y})$ is analytic   (6)

    b)   Evaluate $\int_C \frac{1}{\left(z^3 - 1\right)^2} dz$ where C is the circle $|z - 1| = 1$ (6)

    c)   Show that the set of functions $\left\{ Sin\left(\frac{\pi x}{2L}\right), Sin\left(\frac{3\pi x}{2L}\right), Sin\left(\frac{5\pi x}{2L}\right), \dots \right\}$ (8)
forms an orthogonal set over the interval $[0, L]$. Construct
corresponding orthonormal set.

TURN OVER

4 a) Find Laplace Transform of the periodic function (6)

$$f(t) = \begin{cases} \sin 2t, & 0 < t < \dfrac{\pi}{2} \\ 0, & \dfrac{\pi}{2} < t < \pi \end{cases} \quad f(t) = (t+\pi)$$

b) Find half range sine series for $x \sin x$ in $(0,\pi)$ (6)

c) Expand $f(z) = \dfrac{z^2-1}{z^2+5z+6}$ around $z = 1$ (8)

5 a) Using residue theorem evaluate $\oint_C \dfrac{e^z}{\left(z^2+\pi^2\right)^2}\,dz$ where C is $|z| = 4$ (6)

b) Find Fourier expansion of $f(x) = x + x^2$ in $(-\pi,\pi)$ and $f(x+2\pi) = f(x)$ (6)

c) Find i) $L\left(e^{-4t}\int_0^t u\sin 3u\,du\right)$ ii) $L^{-1}\left(\dfrac{1}{s}\log\left(1+\dfrac{1}{s^2}\right)\right)$ (8)

6 a) Show that the function $w = \dfrac{4}{z}$ transform the straight lines $x = c$ in the z-plane into circles in the w-plane. (6)

b) Solve using Laplace Transform $R\dfrac{dQ}{dt} + \dfrac{Q}{c} = V$, $Q = 0$ when $t = 0$ (6)

c) Solve the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ for the following data by successive iterations (Calculate first two iterations) (8)

0	8.7	12.1	12.8	9.0	
0					17.0
	u_7	u_8	u_9		
0					21.0
	u_4	u_5	u_6		
0					21.9
	u_1	u_2	u_3		
0	11.1	17.0	19.7	18.6	