 14/11/19

SE/Sem III/CBCGS/AUTO/ND-19

(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = e^{-4t}\sin 3t.\cos 2t$. 5

b) Show that the set of functions $f(x) = 1, g(x) = x$ are orthogonal on $(-1,1)$. Determine the constants a and b such that the function $h(x) = -1 + ax + bx^2$ is orthogonal to both $f(x)$ and $g(x)$. 5

c) Evaluate $\int_c (z^2 - 2\bar{z} + 1)dz$ where C is the circle $|z| = 1$. 5

d) Compute the Spearman's Rank correlation coefficient ***R*** and Karl Pearson's correlation coefficient ***r*** from the following data, 5

x	12	17	22	27	32
y	113	119	117	115	121

2. a) Using Laplace transform, evaluate $\int_0^\infty e^{-t}\int_0^t \frac{\sin u}{u}du\,dt$. 6

b) Find an analytic function $f(z) = u + iv$, if 6

$$u = e^{-x}\{(x^2 - y^2)\cos y + 2xy\sin y\}.$$

c) Obtain Fourier series of $f(x) = x^2$ in $(0,2\pi)$. Hence, deduce that – 8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + + \cdots$$

3. a) Using Bender –Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to the conditions,

$u(0,t) = 0, u(4,t) = 0, u(x,0) = x^2(16 - x^2)$ taking $h = 1$, for 3 minutes. 6

b) Using convolution theorem, find the inverse Laplace transform of 6

$$F(s) = \frac{s^2 + s}{(s^2 + 1)(s^2 + 2s + 2)}$$

c) Using Residue theorem, evaluate

i) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ ii) $\int_C \frac{z^2}{(z+1)^2(z-2)}dz$, $C: |z| = 1.5$ 8

4. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0,$ 6
$u(0,t) = 0,\ u(1,t) = 200t,\ u(x,0) = 0$ taking $h = 0.25$ for one-time step.

b) Obtain the Laurent series which represent the function

$f(z) = \frac{4z+3}{z(z-3)(z+2)}$ in the regions, i) $2 < |z| < 3$ ii) $|z| > 3$ 6

c) Solve $(D^2 - 3D + 2)y = 4e^{2t}$ with $y(0) = -3$ and $y'(0) = 5$ where $D \equiv \frac{d}{dt}$ 8

5. a) Find the bilinear transformation under which $1, i, -1$ from the z-plane are mapped onto $0,1,\infty$ of w-plane. 6

b) Find the Laplace transform of
$f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$ and $f(t + 2\pi) = f(t)$. 6

c) Obtain half range Fourier cosine series of $f(x) = x, 0 < x < 2$. Using Parseval's identity, deduce that – 8

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

6. a) Using contour integration, evaluate: 6

$$\int_{-\infty}^{\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9}\,dx$$

b) Using least square method, fit a parabola, $y = a + bx + cx^2$ to the following data, 6

x	-2	-1	0	1	2
y	-3.150	-1.390	0.620	2.880	5.378

c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ under the boundary conditions $u(0,t) = 0,\ u(l,t) = 0,\ u(x,0) = x, (0 < x < l), l$ being the length of the rod. 8
