ME Sem I / EXTC / Choice base / Second half 2018

Q.P. Code: 39197

[Time: Three Hours] [Marks:80]

Please check whether you have got the right question paper.

N.B: 1. Question No.1 is compulsory.
2. Attempt any three questions from remaining five questions.
3. Assume suitable data if necessary and state it clearly.
4. Figures to right indicates full marks.

Q.1. Solve any five **20**

(a) State and explain the four fundamental subspaces for an m x n matrix of rank r.

(b) The distribution function for a random variable X is

$$F(x) = \begin{cases} 1 - e^{-2x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Find i) the density function ii) the probability that X > 2

(c) Define and explain following terms as related to estimators
i)Bias ii) Variance iii) Efficiency iv) Consistency

(d) State and explain the properties of autocorrelation sequence (function)of stationary process.

(e) Let x be a random vector with mean μ_X and autocorrelation R_x. Show that $y = Q^T x$ transforms x to an uncorrelated component vector y if Q is the eigenmatrix of R_x.

(f) State and explain Kalman filtering problem using underlying state variable system.

Q.2. (a) Find column space and null space of matrix **10**

$$A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ -1 & 2 & 0 & 1 \\ 3 & 0 & 1 & 4 \end{bmatrix}$$

(b) Explain Gram-Schmidt orthogonalization procedure and state its applications. **6**

(c) "If A and B are n x n matrices and AB=0, then either A=0 or B=0." Is this a true statement? Explain with example. **4**

Q.3. (a) State the important properties of P SD. Determine the PSD of a zero mean WSS process $x[n]$ with $r_x(l) = a^{|l|}, -1 < a < 1$.

(b) Explain the Kalman filtering algorithm using suitable equations **10**

Q.4. (a) Let $x[n] = A + w[n], n = 0,1,\ldots,N-1$ It is desired to estimate the value of a DC level A in WGN $w[n]$ where $w[n]$ is zero mean and uncorrelated and each sample has variance $\sigma^2 = 7$. Consider the two estimators **10**

i. $\hat{A} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$

ii. $\check{A} = \frac{x[0]+x[N-1]}{2}$

Find the mean and variance of each estimator. State whether these estimators are unbiased . Which one is better according to variance?

(b) Consider the multiple observations x[n]= A+w[n]n=0,1, ... , N -1 where w[n]~N (0, σ^2). Determine the CRLB for A. 10

Q.5 (a) Consider following random processes 10

i) X (t) = A where A is a random variable uniformly distributed between 0 and 1

ii) X[n] = A cos(wn) where A is a Gaussian random variable with mean 0 and variance 1

Determine whether these random processes are WSS or not.

(b) The exponential density function is given by $f_x(x) = e^{-x}u(x)$ where $u(x)$ is a unit step function. Let x1 and x_2 be two IID random variables with exponential pdf. Let $y = x_1 + x_2$ Determine and plot pdf of y 10

Q.6. (a) Consider a linear transformation y = Ax where 10

Find i) Mean vector μ_x ii) Autocorrelation matrix Ry

$$\mu_x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad R_x = \begin{bmatrix} 4 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \\ 2 & 3 \end{bmatrix}$$

(b) Explain application of DKLT (Discrete Karhunen-Loeve Transform) in signal coding using block diagram. Explain scheme for selection of optimal reduced basis. 6

(c) Write a short note on SVD 4

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