2-3/3/2018

Q. P. Code : 30243

(2½ Hours)

[Total Marks: 75]

N. B.: (1) All questions are compulsory.
(2) Make suitable assumptions wherever necessary and state the assumptions made.
(3) Answers to the same question must be written together.
(4) Numbers to the right indicate marks.
(5) Draw neat labeled diagrams wherever necessary.
(6) Use of Non-programmable calculators is allowed.

1. Attempt any three of the following: 15

a. Use the set-roster notation to indicate the elements in each of the following sets.
i. $S = \{n \in Z \mid n = (-1)k$, for some integer $k\}$.
ii. $T = \{m \in Z \mid m = 1 + (-1)i$, for some integer $i\}$.
iii. $U = \{r \in Z \mid 2 \le r \le -2\}$
iv. $V = \{s \in Z \mid s > 2 \text{ or } s < 3\}$
v. $W = \{t \in Z \mid 1 < t < -3\}$

b. Write negations for each of the following statements.
(Assume that all variables represent fixed quantities or entities, as appropriate.)
i. If P is a square, then P is a rectangle.
ii. If today is New Year's Eve, then tomorrow is January.
iii. If n is prime, then n is odd or n is 2.
iv. If x is nonnegative, then x is positive or x is 0.
v. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

c. Determine whether the statements in (i) and (ii) are logically equivalent.
1) Assume x is a particular real number.
i. $x < 2$ or it is not the case that $1 < x < 3$. i. $x \le 1$ or either $x < 2$ or $x \ge 3$.
2) $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

d. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{b, c, e\}$.
i. Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?
ii. Find $(A - B) - C$ and $A - (B - C)$. Are these sets equal?

e. Use an element argument to prove each statement
i. For all sets A, B, and C, $(A - B) \cup (C - B) = (A \cup C) - B$.
ii. For all sets A, B, and C, $(A - B) \cap (C - B) = (A \cap C) - B$.

f. i. Write a negation for each of the following statements. Indicate which is true, the statement or its negation. Justify your answers.
- $\forall$sets S, $\exists$a set T such that $S \cap T = \emptyset$.
- $\exists$a set S such that $\forall$sets T, $S \cup T = \emptyset$.

ii. Verify whether the given statement is True or False
For all sets A, B, and C, $A \cap (B - C) = (A \cap B) - (A \cap C)$.

2. Attempt any three of the following: 15

a. A menagerie consists of seven brown dogs, two black dogs, six gray cats, ten black cats, five blue birds, six yellow birds, and one black bird. Determine which of the following statements are true and which are false.
i There is an animal in the menagerie that is red.
ii. Every animal in the menagerie is a bird or a mammal.

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iii. Every animal in the menagerie is brown or gray or black.
iv. There is an animal in the menagerie that is neither a cat nor a dog.
v. No animal in the menagerie is blue.
vi. There are in the menagerie a dog, a cat, and a bird that all have the same color.

b. Let *D* be the set of all students at your school, and let *M(s)* be "*s* is a math major," let *C(s)* be "*s* is a computer science student," and let *E(s)* be "*s* is an engineering student." Express each of the following statements using quantifiers, variables, and the predicates *M(s)*,*C(s)*, and *E(s)*.
i. There is an engineering student who is a math major.
ii. Every computer science student is an engineering student.
iii. No computer science students are engineering students.
iv. Some computer science students are also math majors.
v. Some computer science students are engineering students and some are not.

c. Prove that √5 is irrational.

d. Prove that for all integers *n*, if $n > 2$ then there is a prime number *p* such that $n < p < n!$.

e. Prove that for all integers *n*, if n^2 is odd then *n* is odd.

f. Prove that every prime number except 2 and 3 has the form $6q + 1$ or $6q + 5$ for some integer *q*.

3. Attempt <u>any three</u> of the following: 15

a. Prove that $n^3 - n$ is divisible by 6, for each integer $n \geq 0$.
b. Prove that $n^3 - 7n + 3$ is divisible by 3, for each integer $n \geq 0$.
c. Find the first four terms of each of the recursively defined sequence
$S_k = S_{k-1} + 2S_{k-2}$, for all integers $k \geq 2$ $S_0 = 1$, $S_1 = 1$
d. Indicate whether the statements in parts (i)–(iv) are true or false. Justify your answers.
i. If two elements in the domain of a function are equal, then their images in the co-domain are equal.
ii. If two elements in the co-domain of a function are equal, then their preimages in the domain are also equal.
iii. A function can have the same output for more than one input.
iv. A function can have the same input for more than one output.

e. Prove or give counter examples for the following
i) If $f: X \to Y$ and $g: Y \to Z$ are functions and $g \circ f$ is one-to-one, must *g* be one-to-one?
ii) If $f: X \to Y$ and $g: Y \to Z$ are functions and $g \circ f$ is onto, must *f* be onto?

f. Prove or give counter examples for the following
1. Define $g: Z \to Z$ by the rule $g(n) = 4n - 5$, for all integers *n*.
(i) Is *g* one-to-one?
(ii) Is *g* onto?
2. Define $G: R \to R$ by the rule $G(x) = 4x - 5$ for all real numbers *x*. Is *G* onto?

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4. Attempt <u>any three</u> of the following: **15**

a.
i) Let $R = \{(0, 1), (0, 2), (1, 1), (1, 3), (2, 2), (3, 0)\}$. Find R\$, the transitive closure of R.
ii) Let $S = \{(0, 0), (0, 3), (1, 0), (1, 2), (2, 0), (3, 2)\}$. Find St, the transitive closure of S.
iii) Let $T = \{(0, 2), (1, 0), (2, 3), (3, 1)\}$. Find Tt, the transitive closure of T.

b. The relation R is an equivalence relation on the set A. Find the distinct equivalence classes of R.
i) A = {0, 1, 2, 3, 4}
R = {(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3),(4, 0), (4, 4)}
ii) A = {a, b, c, d}
R = {(a, a), (b, b), (b, d), (c, c), (d, b), (d, d)}
iii) A = {1, 2, 3, 4, . . . , 20}. R is defined on A as follows:
For all x, y ∈A, x R y ⇔4 | (x − y).

c. Verify the following statements.
i. 128 ≡ 2 (mod 7) and 61 ≡ 5 (mod 7)
ii. (128 + 61) ≡ (2 + 5) (mod 7)
iii. (128 − 61) ≡ (2 − 5) (mod 7)
iv. (128·61) ≡ (2·5) (mod 7)
v. 1282 ≡ 22 (mod 7)

d. Define the following with suitable example
i) Trail
ii) Path
iii) Circuit
iv) Walk
v) Tree

e.
i) Draw all non isomorphic graphs with six vertices, all having degree 2.
ii) Draw four non isomorphic graphs with six vertices, two of degree 4 and four of degree 3.

f. Draw a graph with the given specifications or explain why no such graph exists.
i) Tree, nine vertices, nine edges
ii) Graph, connected, nine vertices, nine edges
iii) Graph, circuit-free, nine vertices, six edges
iv) Tree, six vertices, total degree 14
v) Graph, circuit-free, seven vertices, four edges

5. Attempt <u>any three</u> of the following: **15**

a. How many positive three-digit integers are multiples of 6? What is the probability that a randomly chosen positive three-digit integer is a multiple of 6? What is the probability that a randomly chosen positive three-digit integer is a multiple of 7?

b. The instructor of a discrete mathematics class gave two tests. Twenty-five percent of the students received an A on the first test and 15% of the students received A's on both tests. What percent of the students who received A's on the first test also received A's on the second test?

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c. An urn contains four balls numbered 2, 2, 5, and 6. If a person selects a set of two balls at random, what is the expected value of the sum of the numbers on the balls?

d. i) Given a set of 52 distinct integers, show that there must be 2 whose sum or difference is divisible by 100.
ii) Show that if 101 integers are chosen from 1 to 200 inclusive, there must be 2 with the property that one is divisible by the other.

e. i. How many distinguishable ways can the letters of the word HULLABALOO be arranged in order?
ii. How many distinguishable orderings of the letters of HULLABALOO begin with U and end with L?
iii. How many distinguishable orderings of the letters of HULLABALOO contain the two letters HU next to each other in order?

f. A pool of 10 semifinalists for a job consists of 7 men and 3 women. Because all are considered equally qualified, the names of two of the semifinalists are drawn, one after the other, at random, to become finalists for the job.
i. What is the probability that both finalists are women?
ii. What is the probability that both finalists are men?
iii. What is the probability that one finalist is a woman and the other is a man?
