Q.P. Code:05601

[Time: 21/2Hours]

[Marks:75]

Please check whether you have got the right question paper.

N.B:

- 1. All questions are compulsory.
- 2. Figures to the right indicate marks.





- a) Choose the best choice for the following questions:
 - 1) Let f be defined on an interval, and let x_1 and x_2 be points on the interval, then f is said to be a
 - p) $f(x_1)=f(x_2)=0$
 - q) $f(x_1)=f(x_2)=1$
 - r) $f(x_1)=f(x_2)=k$
 - s) all of these
 - 2) if f''(a) exists and f has an inflection point at x = a, then
 - p) f''(a) > 0
 - q) f''(a) < 0
 - r) f''(a)=0
 - s) none of these
 - 3) If a function f is continuous on an interval [a,b], then which of the following is true:
 - p) f is integrable on [a,b]
 - q) f is differentiable on [a,b]
 - r) Either (P) or (q)
 - s) None of these
 - 4) the graph of a function of two variables is a surface in
 - p) 1-space
 - q) 2-space
 - r) 3-space
 - s) None of theses
 - 5) which of the following is true about the function $f(x,y) = \frac{xy}{1+x^2+y^2}$?
 - p) Continuous everywhere
 - q) Continuous except where 1+x2+y2=0
 - r) Either (p) or (q)
 - s) Neither (p) nor (q)
- b) Fill in the blanks for the following questions:
 - 1. Two non-negative numbers, x and y, have a sum equal to 10. The largest possible product of the two numbers is obtained by maximizing f (x)=----- for x in the interval.
 - 2. If y = f(x) is a smooth curve on the interval [a, b] then the arc length of this curve over [a, b] is defined
 - 3. A solution of a differential equation $\frac{dy}{dx}$ y=0 is given by -----
 - 4. If f (x, y)= $\sqrt{y+1}\log(x^2-y)$, the value of f (e,0) is given by------
 - 5. The value of $\lim (x, y) \rightarrow (3,2)x\cos(\pi y) = \cdots$

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- c) State true or false for the following questions:
 - 1. Newtons Method uses the tangent line to y=f(x) at $x=x_n$ to compute x_n+1 .

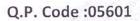
 - The differential equation \$\frac{d^2y}{dx^2} = \frac{dy}{dx}\$ has a solution which is constant.
 If f and g are functions of two variables such that f + g and f g are both continuous, then f and g are themselves continuous.
 - 4. If $f(x,y) \rightarrow L$ as $(x,y) \rightarrow (x_0,y_0)$, then $f(x,y) \rightarrow L$ as $(x,y) \rightarrow (x_0,y_0)$ along any smooth curve.
 - 5. A function f of two variables is said to have an absolute maximum at a point (x_0, y_0) if $f(x_0, y_0) \le f(x_0, y_0)$ y)for all points (x, y) in the domain of f.
- ANSWER ANY three of the following question: Q.2
 - a) Find the intervals on which $f(x)=x^2-4x+3$ is increasing and the intervals on which it is decreasing.
 - b) Use first and second derivative tests to show that $f(x)=x^3-3x+3$ has a relative minimum at x=1 and a relative maximum at x=-1.
 - c) Locate the critical points of $f(x) = 3x^4 + 12x$.
 - d) Find the absolute maximum and minimum values of $f(x) = 4x^2 12x + 10$ in [1,2].
 - e) A firm determines that x units of its product can be sold daily at p Rupees per unit, where x=1000p. The cost of producing x units per day is C(x) = 3000 + 20x.
 - 1. Find the revenue function R(x)
 - 2. Find the profit function P(x)
 - 3. Assuming that the production capacity is at most 500 units per day, determine how many units the company must produce and sell each day to maximize the profit
 - 4. Find the maximum profit.
 - The equation x³-x-1=0 has one real solution. Approximate it by Newtons Method.
- Answer any THREE of the following questions: Q.3
 - a) Find the area under the curve $y=3\sqrt{x}$ over the interval [1,4].
 - b) Find the area of the region enclosed by $x=y^2$ and y=x-2, integrating with respect to y.
 - c) Find the approximate value of $\int_{1}^{2} \frac{1}{x} dx$ using Simpson's rule with n=14.

 - d) Solve differential equation $\frac{dy}{dx} = 2(1 + y^2)x$. e) Use Euler's Method with a step size of 0.5 to find approximate solution of the initial-value problem $\frac{dy}{dx} = y^{\frac{1}{3}}, y(x) = 1$ over $0 \le x \le 4$.
 - f) Solve the differential equation $\frac{dy}{dx} y = e^x$ by the method of integrating factors.

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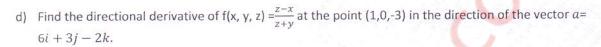


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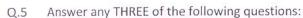
- Answer any THREE of the following question:
 - a) Find $\lim_{(x,y)\to(0,0)} \frac{xy}{x^{2+y^2}}$ (1) along x-axis and (2) along the parabola $y=x^2$.
 - b) Determine whether the limit exists. If so, find its value.

$$\lim_{(x, y)\to(0,0)} \frac{1-x^2-y^2}{x^2+y^2}$$

 $\begin{aligned} & \text{Lim}_{(x,\,y)\to(0,0)} \frac{1-x^2-y^2}{x^2+y^2}. \\ \text{c)} & \text{ Find f}_x(\textbf{2},\textbf{1}) \text{ and f}_y(\textbf{1},\textbf{2}) \text{ for the function f } (x,y) = 10x^2y^4 - 6xy^2 + 10x^2. \end{aligned}$



- e) Find parametric equations of the tangent line to the curve of intersection of the paraboloid $z=x^2+y^2$ And the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point (1,1,2).
- Find all relative extrema and saddle points of f (x,y)=1- x^2-y^2



- a) Let $f(x)=x^2+px+q$. Find the values of p and q such that f(1)=3 is an extreme value of f on [0,2]. Is this value a maximum or minimum?
- b) Show that $y=xe^{\frac{-x^2}{2}}$ satisfies the equation $xy=(1-x^2)y$.
- Find the area of the region below the interval [2,1] and above the curve $y=x^3$
- d) Solve differential equation $\frac{dy}{dx}y = e^{2x}$.
- e) Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{(x^2+y^2)}}$, by converting to polar coordinates.

