FE - SEM II (CBGS)

A.M. - II

11/2/16

QP Code : 28611

(3 hours)

Total Marks: 80

N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

Q.1

(a) Prove that $\int_0^1 \frac{dx}{\sqrt{-\log x}} = \sqrt{\pi}$ [3]

(b) Solve $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = 0$ [3]

(c) Prove that $\Delta\nabla = \nabla\Delta$ [3]

(d) Solve $[xy\sin(xy) + \cos(xy)]y\,dx + [xy\sin(xy) - \cos(xy)]x\,dy = 0$ [3]

(e) Change to polar coordinates and evaluate $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2 + y^2)\,dx\,dy$ [4]

(f) Evaluate $\int_0^1 \int_0^x (x^2 + y^2)\,x\,dy\,dx$ [4]

Q.2

(a) Solve $(1+y^2)dx = \left(e^{\tan^{-1}y} - x\right)dy$ [6]

(b) Change the order of integration and evaluate [6]

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx$$

(c) Prove that $\int_0^\infty \frac{e^{-x} - e^{-ax}}{x\sec x}\,dx = \frac{1}{2}\log\left(\frac{a^2+1}{2}\right)$ [8]

Q.3

(a) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\,dz\,dy\,dx$ [6]

(b) Find the total area of the curve $r = a\sin 2\theta$ [6]

(c) Solve $x^2\frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2\log x$ [8]

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Q.4 (a) Show that the length of the arc of the curve $a\,y^2 = x^3$ from the origin to the point whose abscissa is b is $\frac{8a}{27}\left[\left(1+\frac{9b}{4a}\right)^{3/2}-1\right]$ [6]

(b) Solve $\left(D^2 - D - 2\right)y = 2\log x + \frac{1}{x} + \frac{1}{x^2}$ [6]

(c) Apply Runge-kutta Method of fourth order to find an approximate value of y for $\frac{dy}{dx} = x\,y$ with $x_0 = 1, y_0 = 1$ at $x = 1.2$ taking $h = 0.1$ [8]

Q.5 (a) Solve $\left(x^2 y - 2xy^2\right)dx - \left(x^3 - 3x^2 y\right)dy = 0$ [6]

(b) Using Taylor series Method obtain the solution of following differential equation $\frac{dy}{dx} = 2y + 3e^x$ with $y_0 = 0$ when $x_0 = 0$ for $x = 0.1, 0.2$ [6]

(c) Find the approximate value of $\int_0^4 e^x\,dx$ by i) Trapezoidal Rule , ii) Simpson's 1/3rd Rule [8]

Q.6 (a) In a circuit containing inductance L, resistance R, and voltage E, the current I is given by $L\frac{di}{dt} + Ri = E$. Find the current i at time t if at $t = 0, i = 0$ and L, R, E are constants. [6]

(b) Evaluate $\iint_R \frac{dx\,dy}{\left(1+x^2+y^2\right)^2}$ over one loop of the lemniscate $\left(x^2+y^2\right)^2 = x^2 - y^2$ [6]

(c) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$ [8]

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