Q.P. Code :16282

Max. Marks: 80 **Time: 3 hrs.**

N.B. : 1. Q1 is compulsory
2. Attempt any three questions from Q2 to Q6.

Q1. a) Evaluate $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\,dx$ 3

b) Solve $(D^3+1)^2 y = 0$ 3

c) Solve the ODE $\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \left(x + xy^2\right)dy = 0$ 3

d) Use Taylor's series method to find a solution of $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$ at $x = 0.1$ taking $h = 0.1$ correct to three decimal value. 3

e) Given $\int_0^x \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$, using DUIS find the value of $\int_0^x \frac{dx}{\left(x^2+a^2\right)^2}$ 4

f) Find the perimeter of the curve $r = a(1-\cos\theta)$. 4

Q2. a) Solve $(D^3 + D^2 + D + 1)y = \sin^2 x$ 6

b) Change the order of integration $\int_0^a \int_{\sqrt{a^2-x^2}}^{x+3a} f(x,y)\,dx\,dy$ 6

c) Evaluate $\iint_R \frac{2xy^5}{\sqrt{1+x^2y^2-y^4}}\,dx\,dy$, where R is a triangle whose vertices are $(0,0),(1,1),(0,1)$. 8

Q3. a) Find the volume enclosed by the cylinder $y^2 = x$ & $y = x^2$ cut off by the planes $z = 0$, $x + y + z = 2$. 6

b) Using Modified Euler's method, find an approximate value of y at $x = 0.2$ in two step taking $h = 0.1$ and using three iteration, given that $\frac{dy}{dx} = x + 3y$, $y = 1$ when $x = 0$. 6

c) Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$ 8

Q4. a) Show that $\int_0^a \sqrt{\frac{x^3}{a^3-x^3}}\,dx = \frac{a\sqrt{\pi}\,\Gamma(5/6)}{\Gamma(1/3)}$ 6

b) Solve $(D^2+2)y = e^x\cos x + x^2 e^{3x}$ 6

Q.P. Code :10

c) Use polar co-ordinates to evaluate $\iint \frac{(x^2+y^2)^2}{x^2 y^2}\, dx\, dy$ over the area 8

common to the circle $x^2 + y^2 = ax$ and $x^2 + y^2 = by$, $a > b > 0$.

Q5. a) Solve $y\,dx + x\left(1 - 3x^2 y^2\right) dy = 0$ 6

b) Find the mass of a lamina in the form of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if the 6

density at any point varies as the product of the distance from the axes of the ellipse.

c) Compute the value of $\int_0^{\pi/2} \sqrt{\sin x + \cos x}\, dx$ using (i) Trapezoidal rule 8

(ii) Simpson's $(1/3)^{rd}$ rule (iii) Simpson's $(3/8)^{th}$ rule by dividing into six subintervals.

Q6. a) Evaluate $\iiint_V x^2\, dx\, dy\, dz$ over the volume bounded by the planes 6

$x = 0,\ y = 0,\ z = 0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

b) Change the order of integration and evaluate $\int_0^2 \int_{\sqrt{2y}}^2 \frac{x^2}{\sqrt{x^4 - 4y^2}}\, dx\, dy$ 6

c) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ 8