1. FE) sem. I., CBSGS (old), FH2018 14/05/18

App Maths.- I

Q. P. Code: 37722

(3 hours) **[Total Marks: 80]**

Note: i) Q.No 1 is compulsory
ii) Attempt any three from remaining.
ii) All questions carry equal marks.

Q.No.1) a)If tanhx = $\frac{2}{3}$, find the value of x and cosh2x. (3)

b) If u = $e^{ax}sinby$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ (3)

c) If $x = ucosv$, $y = usinv$, Prove that $JJ' = 1$ (3)

d) Prove that $\log(1 + sinx) = x - \frac{x^2}{2} + \frac{x^3}{6} + \cdots$ (3)

e) Show that every skew Hermitian matrix A can be uniquely expressed as P+iQ, where P is real skew symmetric & Q is real symmetric. (4)

f) Find nth derivative of $2^x sin^2 xcosx$ (4)

Q.No.2) a) If $tanz = \frac{1}{2}(1 - i)$ prove that $z = \frac{1}{2}tan^{-1}2 + \frac{i}{4}log\left(\frac{1}{5}\right)$ (6)

b) Reduce the matrix to normal form hence find its rank $\begin{bmatrix} 2 & -4 & 3 & 0 \\ 1 & -2 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 4 & -7 & 4 & 5 \end{bmatrix}$ (6)

c) If u = $sin^{-1}\left\{\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}\right\}$, show that (8)

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial xy} + y^2\frac{\partial^2 u}{\partial x^2} = \frac{tanu}{400}(tan^2 u - 19)$$

Q.No 3) a) Test the consistency and if possible solve the equations

2x – y - z = 2, x + 2y + z = 2, 4x - 7y - 5z = 2 (6)

b) Find all the stationary values of $x^3+3xy^2-15x^2-15y^2+72x$ (6)

c) If coshx = secθ, Prove that

(i) x = log (secθ + tanθ) (ii) $\theta = \frac{\pi}{2} - 2tan^{-1}(e^{-x})$ (8)

Q.No.4) a) If $u = xy^2 + e^{xy^2}$ Prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ (6)

b) Prove that $Log\left[\frac{(a-b)+i(a+b)}{(a+b)+i(a-b)}\right] = i\left(2n\pi + tan^{-1}\frac{2ab}{a^2-b^2}\right)$ (6)

c) Solve the following system of equation by Jacobi method (8)

4x+y+3z =17, x+5y+z =14, 2x-y+8z = 12

Q.No.5) a) Show that $\cos^6\theta - \sin^6\theta = \frac{1}{16}[\cos 6\theta + 15\cos 2\theta]$ (6)

b) Find a and b such that $\lim_{x\to 0} \frac{a\sin^2 x + b\log\cos x}{x^4} = \frac{1}{2}$ (6)

c) If y = cos(mlogx), show that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$$ (8)

Q.No.6) a) Are the vectors X_1 =[1,3,4,2], X_2 =[3,-5,2,6] , X_3 =[2,-1,3,4] linearly dependent?

If so, express X_1 as a linear combination of the others (6)

b) If $f(x^2y^3,\ z - 3x) = 0$, Prove that $3x\frac{\partial z}{\partial x} - 2y\frac{\partial z}{\partial y} = 9x$ (6)

c) Fit a second degree parabola to the following data (8)

x	-2	-1	0	1	2
y	1	1.8	1.3	2.5	6.3

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