F E(Sem-I) / CBGS(R-12) / A-Maths-I / May-17

Q. P. Code: 13970

(3 Hours) Total Marks :80

16.05.2017

N.B. (1)Question No. 1 is compulsory

(2) Solve any Three from the remaining.

(3) Figure in right indicates full marks.

K. G. C. E. EXAM KARJAT, RAIGAD

Q.1)(a) Prove that $\dfrac{1}{1-\dfrac{1}{(1-\sec h^2 x)}} = -\sinh^2 x$ 3

(b)If $z(x+y)=(x^2+y^2)$ Prove that $\left(\dfrac{\partial z}{\partial x}-\dfrac{\partial z}{\partial y}\right)^2 = 4\left(1-\dfrac{\partial z}{\partial x}-\dfrac{\partial z}{\partial y}\right)$ 3

(c)If $u = e^x \cos y, v = e^x \sin y$ find $\dfrac{\partial(u,v)}{\partial(x,y)}$ 3

(d)Using Maclaurin's series prove that $e^x \log(1+x) = x + \dfrac{x^2}{2!} + \dfrac{2x^3}{3!} + \ldots\ldots$ 3

(e)Prove that the matrix $\dfrac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is Unitary. 4

(f)Find the nth Derivative of $y = \cos x \cos 2x \cos 3x$ 4

Q.2) (a)Find the roots common to $x^4+1=0$ and $x^6-i=0$ 6

(b)Find the nonsingular matrices P and Q such that PAQ is in Normal Form . Also find rank of A 6

Where $A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

(c)State and Prove Eulers theorem for Homogeneous functions on two variables and hence 8

find the value of $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$ if $u=\dfrac{1}{x^2}+\dfrac{1}{xy}+\dfrac{\log x-\log y}{x^2+y^2}$

Q.3(a)Determine the values of K for which the following equations are consistent. Also solve 6

the system for these values of k $x+2y+z=3, x+y+z=k, 3x+y+3z=k^2$

(b)Find the stationary values of $f(x,y)=y^2+4xy+3x^2+x^3$ 6

(C)If $\tan(\alpha+i\beta)=\cos\theta+i\sin\theta$ Prove that $\alpha=\dfrac{n\pi}{2}+\dfrac{\pi}{4}$ and $\beta=\dfrac{1}{2}\log\tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{2}\right)$ 8

Turn Over

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Q.4(a)If $u=\log\left(x^3+y^3+z^3-3xyz\right)$ Prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$ 6

(b)Prove that principal value of $(1+\tan\alpha)^{-i}$ is $e^{\alpha}\left[\cos(\log\cos\alpha)+i\sin(\log\cos\alpha)\right]$ 6

(C)Apply Crouts method to solve system of linear Equations $x+y+z=3,\ x+2y+3z=4,\ x+4y+9z=6$ 8

Q.5(a)Find the constants a,b,c so that $\lim_{x\to 0}\frac{x(a+b\cos x)-c\sin x}{x^5}=1$ by L'Hospitals rule 6

(b)Prove That $e^{e^x}=e\left[1+x+x^2+\frac{5}{6}x^3+\frac{5}{8}x^4+........\right]$ 6

(c)If $x=\tan(\log y)$ prove that $(1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1)y_{n-1}=0$ 8

Q.6(a)If $A=\begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal find a,b,c. 6

(b)Fit a straight line y=a+bx to the following data 6

x	1	2	3	4	5	6
y	49	54	60	73	80	86

(C)If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1,$ prove that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$

Given that u is an implicit function of x,y,z. 8