Sem - I / CBCGS / Applied Maths - I | 07-12-2016

Q.P. Code : 803104

(3 Hours) **[Total Marks : 80**

N.B : (1) Question No.1 is compulsory.
(2) Answer any three questions from remaining.
(3) Assume suitable data if necessary.

1. (a) If $\cos\alpha\cosh\beta = \frac{x}{2}, \sin\alpha\sinh\beta = \frac{y}{2}$, Prove that 03

$$\sec(\alpha - i\beta) + \sec(\alpha + i\beta) = \frac{4x}{x^2 + y^2}$$

(b) If $z = \log(e^x + e^y)$, show that $rt - s^2 = 0$, where 03

$$r = \frac{\partial^2 z}{\partial x^2}, t = \frac{\partial^2 z}{\partial y^2}, s = \frac{\partial^2 z}{\partial x \partial y}$$

(c) If $x = uv$, $y = \frac{u+v}{u-v}$. Find $\frac{\partial(u,v)}{\partial(x,y)}$. 03

(d) If $y = 2^x \sin^2 x \cos x$ find y_n 03

(e) Express the matrix 04

$$A = \begin{bmatrix} 1 & 0 & 5 & 3 \\ -2 & 1 & 6 & 1 \\ 3 & 2 & 7 & 1 \\ 4 & -4 & 2 & 0 \end{bmatrix}$$ as the sum of symmetric and skew-symmetric matrices.

(f) Evaluate $\lim_{x \to 0} \frac{e^{2x} - (1+x)^2}{x\log(1+x)}$ 04

2. (a) Show that the roots of $x^5 = 1$ can be written as $1, \alpha, \alpha^2, \alpha^3, \alpha^4$ .Hence show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$ 06

(b) Reduce the following matrix to its normal form and hence find its rank. 06

$$A = \begin{bmatrix} 3 & -2 & 0 & 1 \\ 0 & 2 & 2 & 7 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

(c) Solve the following system of equations by Gauss-Seidel Iterative Method upto four iterations. 08

$4x - 2y - z = 40$

$x - 6y + 2z = -28$

$x - 2y + 12z = -86$

TURN OVER

3. (a) Investigate for what values of 'λ' and 'μ' the system of equations 06

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

has (i) no solution
(ii) a unique solution
(iii) an infinite no. of solutions.

(b) If $u = x^2 + y^2 + z^2$, where $x = e^t, y = e^t \sin t, z = e^t \cos t$ 06

Prove that $\frac{du}{dt} = 4e^{2t}$

(c) i) Show that $\sin(e^x - 1) = x + \frac{x^2}{2} - \frac{5x^4}{24} + \ldots\ldots$ 04

ii) Expand $2x^3 + 7x^2 + x - 6$ in powers of $x - 2$ 04

4. (a) If $x = u + v + w, y = uv + vw + uw, z = uvw$ and ϕ is a function of x, y and z. 06
Prove that

$$x\frac{\partial \phi}{\partial x} + 2y\frac{\partial \phi}{\partial y} + 3z\frac{\partial \phi}{\partial z} = u\frac{\partial \phi}{\partial u} + v\frac{\partial \phi}{\partial v} + w\frac{\partial \phi}{\partial w}$$

(b) If $\tan(\theta + i\phi) = \tan\alpha + i\sec\alpha$, 06

Prove that i) $e^{2\phi} = \cot\frac{\alpha}{2}$ ii) $2\theta = n\pi + \frac{\pi}{2} + \alpha$

(c) Find the root of the equation $x^4 + x^3 - 7x^2 - x + 5 = 0$ which lies between 2 and 2.1 correct to three places of decimals using Regula Falsi Method. 08

5. (a) If $y = \left(x + \sqrt{x^2 - 1}\right)^m$, Prove That 06

$$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0 .$$

(b) Using the encoding matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, encode and decode the message 06

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(c) i) Considering only principal values separate into real and imaginary parts 04

$i^{\log(1+i)}$

ii) Show that $i\log\left(\frac{x-i}{x+i}\right) = \pi - 2\tan^{-1}x$ 04

TURN OVER

6. (a) Using De Moivre's theorem prove that 06

$$\cos^6\theta - \sin^6\theta = \frac{1}{16}(\cos 6\theta + 15\cos 2\theta)$$

(b) If $u = \sin^{-1}\left(\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}}\right)^{\frac{1}{2}}$, Prove that 06

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}\left(\tan^2 u + 13\right)$$

(c) Discuss the maxima and minima of $f(x,y) = x^3y^2(1-x-y)$ 08